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Rational Altruism, incentives in teams, equity based pay system, wicked masked altruism.

JEL codes:
D64, M21, D23.

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In the last several years, many organisations have been experimenting with team-based production. The formation of teams is economically desirable when they lead to possible gains from complementarities in production among workers, facilitate gains from specialisation and/or encourage gains from the transfer of information which might be valuable to other partnerships (Lazear, 1998).

In this context, many firms have implemented team-type incentive systems in order to discourage opportunistic behaviour by their members (the free-rider problem). The specialised literature has therefore focused on providing different alternatives aimed at solving such inefficiencies.

On the one hand are the hierarchy-based solutions, largely consisting of including an additional agent to perform supervisory (Alchian and Demsetz 1972) or administration functions (Holmström 1982). Both these proposals have associated management costs in the residual income paid to the additional agent joining the organisation. They both, however, reach Nash equilibrium in which individual resources (or effort) have zero residual loss.

On the other hand are the non-hierarchical solutions which analyse the role played by cultural elements (e.g. peer pressure, sociability, solidarity) in the appearance of cooperative conduct among team members. Within the possible range of solutions, this paper focuses on the role played by the appearance of altruistic conduct among co-workers.

Altruism is a complex concept which has been analysed in different fields such as sociology or psychology. The alternative contemplated here is to use economics as a basic analytical discipline. Indeed, Simon (1993) claims that the social and business worlds contain examples of altruistic conduct which can and should be analysed by economists.

From an economic perspective, Deckop (1995: 359) defines altruism “as the self-initiated desire to work for the benefit of others, without expectation of external rewards sufficient to justify the desire”. The author distinguishes between organisational and
societal components. Organisational altruism results from the moral commitment of individuals to the organisation. Societal altruism results from sources other than the organisation. The intended beneficiary of organisational altruism is the organisation itself, whereas societal altruism only indirectly has a positive impact on the organisation. This type of altruism corresponds to the sociability concept defined by Goffee and Jones (1996: 134): “it is the measure of emotional, non-instrumental relations (those in which people do not see others as a means of satisfying their own ends) among individuals who regard one another as friends”. Altruism, therefore, is seen as an exogenous characteristic of individuals which alters their preference function so that the individual utility of an agent driven by solidarity is increased as the utility of his colleagues grows. This leads to cooperative behaviour among team members which can increase business efficiency.

The appearance of altruistic feelings among team members, however, does not necessarily depend on the existence of exogenous feelings of solidarity between them. Rotemberg (1994) shows how completely selfish agents can have an incentive to show solidarity in order to enhance their own material utility. As a result, the Nash level of effort of all the team members is greater than their respective Nash level when the possibility of such rational or self-serving altruistic feelings is not considered. However, Rotemberg also shows that the role played by this type of altruism never leads to a Pareto-optimal solution. We are therefore referring to endogenous or strategic altruism, known as Rational Altruism.

Based on the work of Rotemberg, this paper performs an in-depth analysis of team members’ motivations to be rationally altruistic, and the effect of such altruism on business efficiency.

Our analyses show the existence of an additional component of rational altruism, other than the purely strategic aspect suggested by Rotemberg, which we have called Wicked Masked Altruism. This component has a negative impact on business efficiency when there are incentives for its appearance.

In order to prevent these negative effects derived from the additional component detected in rational altruism, we propose a remuneration system based on total output which enables us to limit the parameters of rational altruism to the range of values initially postulated by Rotemberg with the corresponding increase in efficiency.
In order to clearly present these findings, we devote Section I to presenting Rational Altruism in the terms used by Rotemberg (1994). In section II, we analyse the values of the parameters of rational altruism when agents are not homogeneous (different productivities). In section III, we design a remuneration system which fosters group efficiency in collective terms. In Section IV, from the results obtained in sections II and III, we identify the existence of an additional component in rational altruism other than the purely strategic factor identified by Rotemberg. Finally, Section V contains our main conclusions and their implications.

I. Rotemberg altruism revisited

Rotemberg (1994) introduces rational altruism when analysing what motivates an organisation’s workers to be altruistic. Rational altruism is the manifestation that an agent is concerned with the welfare of others, even if he is actually selfish. The welfare of an agent with no solidarity can therefore be greater than if he acts selfishly. It is, therefore, an example of strategic, endogenous or self-serving altruism. In this context, Rotemberg (1994) shows that the rational altruism parameters ranges from 0 to 1 ($0 < \lambda_i < 1$) if the effort added to the group is complementary and the marginal utility obtained by agent $i$ by increasing his effort ($a_i$) is in turn enhanced by the greater efforts of the other agent ($a_j$).

In order to illustrate Rotemberg’s contribution and its implications, we formulated a model to meet the specifications established by the author.

We selected a Cobb-Douglas production function (Cobb and Douglas, 1928)\(^2\) with decreasing returns to scale. It accounts for the presence of team technology/production and the existence of complementarities among productive resources. The collective or team production function, therefore, takes the following form:

$$Y = F(a_1, ..., a_N) = \beta \times \prod_{i=1}^{N} a_i^{\alpha_i}; \quad 0 < \alpha_i < 1, \quad \sum_{i=1}^{N} \alpha_i < 1 \quad (1)$$

$$\beta = 1$$

Where $a_i$ is the specific allocation of resources of each individual ($i = 1, ..., N$) to a

\(^2\) The Cobb-Douglas production function is the simplest example of technology with regular isoquants and it is broadly accepted in economic literature (Varian, 1999; Nicholson, 2004). Moreover, Rotemberg (1994: 699) indicates that in those teams whose production respond to the use of the technology Cobb-Douglas, the coworkers will have incentives to show rational altruism.
given collective action \((a_i \in A_i)\); in other words, its shows the level of resources or effort provided by each agent. On the other hand, \(\alpha_i\) measures the response of the quantity produced to variations in the productive factors or degree of effort\(^3\) of each agent. Finally, parameter \(\beta\) is an approximation to the scale of production, or production volume, obtained when using one unit of each factor (Varian, 1999). To simplify, parameter \(\beta\) takes a value of one.

Each individual or team member supports an opportunity cost for being a member of the team \((C_i(a_i))\). This opportunity cost is the value of the alternative use of the effort which each agent provides for the collective action. It is defined through a growing linear function such as \(C_i(a_i) = \omega_i \times a_i\) where \(\omega_i\) is the market price of each unit of effort (opportunity salary). This growing cost function and the existence of decreasing returns to scale in the production function ensure the existence of a single peak in the total wealth function generated by the organisation (production value less the opportunity cost of the resources required).

Finally, we define the rule for distributing the total output among team members, so that it is a production function of the group\(^4\) \((R_i(Y))\). In particular, we selected an equalitarian distribution among all the members \((R_i(Y) = (Y/N))\).

The use of an equality-based pay system enables us to assume that management costs are zero or negligible, as the variable on which each team member’s remuneration is based is the organisation’s total production, which is easily observable (at least by those involved in the collective action). On the other hand, the use of an equality-based pay system enables greater cohesion among the group, with a positive impact on the performance of individual team members (Lazear, 1989).

In this context, each agent chooses his degree of effort in the team \((a_i)\) with no external constraints and maximising his own material utility.

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\(^3\) To simplify, we denote \(\alpha_i\) as elasticities of the respective productive factors or degree of effort of each agent \((a_i)\).

\(^4\) In a collective action, the only variable which can be measured at a low cost is total production \((Y)\). And the presence of complementarities makes it difficult to measure the part of production which corresponds to each agent. On the other hand, remuneration cannot be according to the effort made by each agent, as the supervisory figure proposed by Alchian and Demsetz (1972) has not been introduced.
\[
\text{Max}_{a_i} \left[ U_i = R_i(Y) - C_i(a_i) = \frac{1}{N} \times \left( \prod_{i=1}^{N} a_i^{a_i} \right) - \sigma_i \times a_i \right] \quad \forall \ i
\] (2).

constrained to \( R_i(Y) \geq C_i(a_i), \quad \forall \ i \)

The degrees of effort of each agent (\( a_i^* \)) which satisfy first order conditions:

\[
\frac{\partial U_i}{\partial a_i} = \frac{dR_i}{dY} \frac{\partial Y}{\partial a_i} - \frac{dC_i}{da_i} = \frac{\partial}{\partial a_i} \left( \frac{1}{N} \times \left( \prod_{i=1}^{N} a_i^{a_i} \right) - \sigma_i \times a_i \right) = 0, \quad i = 1, \ldots, N
\] (3)

are expressed as follows:

\[
a_i^* = \left( \frac{\alpha_i}{N \times \sigma_i} \right)^{1/\sum_{i=1}^{N} a_i} \prod_{j=1}^{N} \left( \frac{\alpha_j}{N \times \sigma_j} \right)^{1/\sum_{i=1}^{N} a_i}
\] (4)

Each agent chooses a degree of effort (\( a_i^* \)) considering those of the other agents (\( a_j^* \), \( \forall i \neq j \)), as a result of the existence of team technology. There is, therefore, a latent coordination problem which is solved by a process of mutual adjustment until a self-binding solution is reached, specifically a Nash equilibrium. For simplicity reasons, we assume that this adjustment is achieved by a mutual process comprising simultaneous decisions repeated over time.

The wealth created by the team (\( WCT^* \)) thus adopts the following expression:

\[
WCT^* = \prod_{i=1}^{N} (a_i^*)^{\alpha_i} - \sum_{i=1}^{N} (\sigma_i \times a_i^*)
\] (5)

The above solution, however, is not Pareto optimal in terms of wealth generated by the organisation. Indeed, the efforts required from each of the agent to obtain the highest possible degree of wealth created (\( WCT^{**} \)), which represents absolute efficiency, is obtained by solving the following problem:

\[
WCT^{**} \equiv \text{Max} \left( F(a_1, \ldots, a_N) - \sum_{i=1}^{N} C_i(a_i) \right) = \text{Max}_{a_i} \left( \prod_{i=1}^{N} a_i^{a_i} - \sum_{i=1}^{N} (\sigma_i \times a_i) \right)
\] (6)

s.a. \( a_i \in A_i \)
Thus, the expression of the efficient level of effort, in collective terms, of each agent is \( a_i^{**} \):

\[
a_i^{**} = \left( \frac{\alpha_i}{\sigma_i} \right)^{1 - \frac{1}{\sum_{i=1}^{N-1} a_i}} \prod_{j=1}^{N-1} \left( \frac{\alpha_j}{\sigma_j} \right)^{1 - \frac{1}{\sum_{i=1}^{N-1} a_i}}
\]

(7)

The \( a_i^{**} \) values are the Nash equilibrium solution. The coordination process through which the agents reach this solution is achieved by a process of mutual adjustment based on simultaneous decisions repeated over time.

Transferring these efficient effort levels to the wealth generation function, we obtain the expression of potential wealth \( WCT^{**} \):

\[
WCT^{**} = \prod_{i=1}^{N} \left( a_i^{*} \right)^{\alpha_i} - \sum_{i=1}^{N} (\sigma_i \times a_i^{**})
\]

(8)

From expressions \( a^* [4] \) and \( a^{**} [7] \), we see that \( a^* < a^{**} \) providing the number of team members is two or more. The wealth created by the team, therefore, is always less than the possible maximum, a result consistent with Holmström’s Theorem (Holmström, 1982). The reason lies in the existence of positive transaction costs (a residual loss derived from moral hazard).

From the expressions \( WCT^* [5] \) and \( WCT^{**} [8] \), we obtain the expression of the residual loss (RL). This shows the inefficiency derived from the existence of diverging objectives among the parties:

\[
RL = WCT^{**} - WCT^* = \prod_{i=1}^{N} \left( a_i^{**} \right)^{\alpha_i} - \sum_{i=1}^{N} (\sigma_i \times a_i^{**}) - \left( \prod_{i=1}^{N} \left( a_i^{*} \right)^{\alpha_i} - \sum_{i=1}^{N} (\sigma_i \times a_i^{*}) \right) =
\]

\[
= \prod_{i=1}^{N} \left[ \left( 1 - \left( \frac{1}{N} \right)^{\frac{a_i}{\sum_{i=1}^{N-1} a_i}} \right) \times \left( \frac{\alpha_i}{\sigma_j} \right)^{1 - \frac{1}{\sum_{i=1}^{N-1} a_i}} \right] - \sum_{i=1}^{N} \left( \sigma_i \times \left[ \left( 1 - \frac{1}{N} \right)^{\frac{a_i}{\sum_{i=1}^{N-1} a_i}} \times \left( \frac{\alpha_i}{\sigma_j} \right)^{1 - \frac{1}{\sum_{i=1}^{N-1} a_i}} \right] \right)
\]

(9)

Finally, we analyse the wealth created by the team when there is a possibility of the
manifestation of self-serving altruistic feelings by team members.

A selfish team member (i) will show self-serving altruistic feelings towards another agent (j) if he transfers the conviction to the other party, agent j, that his utility ($U_{ij}^{R,i}$) is:

$$U_{ij}^{R,i} = U_i(a_i, a_j) + \lambda_i \times U_j(a_j, a_i)$$

(If we apply this to our model) =

$$= \frac{1}{N} \times \prod_{i=1}^{N} \alpha_i - (\sigma_i \times a_i) + \lambda_i \times \sum_{j \neq i} \frac{1}{N} \times \prod_{i=1}^{N} \alpha_j - (\sigma_j \times a_j)$$

(10)

In the previous expression, $U_i(a_i, a_j)$ represents the utility of agent i according to his contributions ($a_i$) and those of agent j ($a_j$); $U_j(a_j, a_i)$ represents the utility of agent j according to his contributions ($a_j$) and those of agent i ($a_i$); and parameter $\lambda_i$ represents how agent i shows that his utility is affected by that of agent j. When parameter $\lambda_i$ is zero, agent i behaves as though he/she was selfish; when the parameters has values of over zero, agent i shows solidarity; and when the parameter is negative, agent i behaves as though he/she was wicked.

Therefore, in expression $U_{ij}^{R,i}$, if agent i can enhance his material welfare ($U_i(a_i, a_j)$) by choosing $\lambda_i > 0$ instead of $\lambda_i = 0$, parameter $\lambda_i$ would represent Rational Altruism. Consequently, if $\lambda_i > 0$, agent i will act towards agent j as if showing solidarity, even though he is actually selfish.

The value of each agent’s Rational Altruism parameters is calculated by backwards induction. The expressions of the individual efforts are first deduced (assuming given Rational Altruism parameters) and the specific expressions of the Rational Altruism parameters are then calculated.

Therefore, each agent will maximise his own utility, considering the possible presence of Rational Altruism:

$$\max_{a_i} U_i^{R,i} = U_i(a_i, a_{-i}) + \lambda_i \times \sum_{j \neq i} U_j(a_j, a_{-j})$$

(11)

In the previous expression [11], if $\lambda_i$ is equal to one for all agents i, the objective function would be to maximise the wealth created by the team, with them all supplying an

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5 We assume that each agent shows the same rational altruism for all team members.
efficient effort in collective terms ($a^{**}$).

If we apply this to our model, the expression [11] would be:

$$\text{Max} U^{Ra}_{i} = \left[ \frac{1}{N} \times \prod_{j=1}^{N} a^{a_j} - (\alpha_i \times a_i) \right] + \lambda_i \times \sum_{j=1}^{N} \left[ \frac{1}{N} \times \prod_{j=1}^{N} a^{a_j} - (\alpha_j \times a_j) \right]$$

(12)

For given values of $\lambda_i$, the Nash levels of effort contributed by each agent to the collective action ($a^{Ra}_{i}$), resolving expression (12), are:

$$a^{Ra}_{i} = \left( \frac{1 + \lambda_i (N-1) \times \alpha_i}{N \times \alpha_i} \right)^{\sum_{j=1}^{N} a_j} \prod_{j=1}^{N} \left( \frac{1 + \lambda_j (N-1) \times \alpha_j}{N \times \alpha_j} \right)^{a_j}$$

(13)

As we have mentioned earlier, Rotemberg (1994) shows that the Rational Altruism parameter ranges from 0 to 1 ($0 < \lambda_i < 1$) if the efforts contributed to the collective action are complementary and the marginal utility obtained by agent $i$ by increasing his effort ($a_i$) is in turn increased by a greater effort by the other agent ($a_j$). The conditions defined by Rotemberg are met in our model. Moreover, we see that, for all values of $\lambda$ within the range determined by Rotemberg, the $a^{Ra}_{i}$ values are always greater than the values of $a^{*}_i$ [4]. This confirms that self-serving manifestations of altruism (Rational Altruism) have a positive impact on business efficiency. However, as the values of the $\lambda$ of all the agents are never equal to one, the role played by rational altruism will never lead to a Pareto efficient solution.

II. Rational altruism with an equality-based pay system between heterogeneous coworkers

The purpose of this section is to show that, in the presence of heterogeneous agents, the limits established by Rotemberg for the Rational Altruism parameter are not met. We first calculate the expression of the levels of Rational Altruism shown by each

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6 'Heterogeneous agents' refers to the non-equality of the elasticity of the effort provided by each agent to the collective action. In other words, there is at least one $\alpha_j$ different from the rest of the agents. For simplicity reasons we do not analyse the effects of asymmetric opportunist costs.
agent. From these expressions, we analyse the derived incidence and repercussions on business efficiency.

Based on the expression representing the effort contributed by each agent to the team in the presence of rational altruism [13], each agent chooses the degree of solidarity he wishes to show, given his foreseen degree of effort. Each agent thus maximises his individual utility:

\[
\max_{\lambda_i} U_i = \frac{1}{N} \times \left( \prod_{i=1}^{N} (a_i^{\text{R.e}})^{\alpha_i} \right) - \sigma_i \times a_i^{\text{R.e}} 
\]

(14)

The values of \(\lambda_i\) which solve the problem (Nash equilibrium) are:

\[
\lambda_i = \frac{\sum_{j=1}^{N-1} \alpha_j}{1 - \sum_{j=1}^{N-1} \alpha_j} \quad \text{(15)}
\]

From expression [15], it can be concluded that the agents participating in the coalition have incentives to show self-serving solidarity (\(\lambda_i > 0\)), as the elasticities of the effort of each of the agents (\(\alpha_i\)) are positive by definition. Furthermore, the existence of Rational Altruism derives in an increase in the organisation’s efficiency, as each agent’s equilibrium level of effort is greater than when the possibility of manifestations of Rational Altruism among the agents is not considered. This result confirms the results obtained by Rotemberg (1994). On the other hand, as the number of agents involved in the collective action grows, there are less self-serving manifestations of Rational Altruism.

Nonetheless, when the team members are heterogeneous, there are economically feasible values of \(\alpha_j\) giving a value of \(\lambda_i\) which is greater than 1. Specifically, team members for whom the elasticities of their effort are less than the arithmetic mean of the elasticities of the other agents, will present a Rational Altruism parameter of more than one (see some examples in table 1). Expressed mathematically, co-worker i will present a Rational Altruism parameter of more than one providing that:

\[
\frac{\sum_{j=1}^{N-1} \alpha_j}{1 - \sum_{j=1}^{N-1} \alpha_j} > \lambda_i > 1
\]
\[
\left( \frac{\sum_{j=1}^{N-1} \alpha_j}{N-1} > 1 - \sum_{j=1}^{N-1} \alpha_j > \alpha_i \right)
\]

(16)

Table 1: Values of Rational Altruism parameters for a case with two agents (coalition).

<table>
<thead>
<tr>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\lambda_1$</th>
<th>$\lambda_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.51</td>
<td>1.04</td>
<td>0.11</td>
</tr>
<tr>
<td>0.10</td>
<td>0.80</td>
<td>4.00</td>
<td>0.11</td>
</tr>
<tr>
<td>0.31</td>
<td>0.65</td>
<td>1.86</td>
<td>0.45</td>
</tr>
<tr>
<td>0.48</td>
<td>0.51</td>
<td>1.04</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Indeed, the constraint established by Rotemberg (1994) in relation to parameter $\lambda_i$ ($0 < \lambda_i < 1$) is only verified if the elasticities of the effort of all the agents participating in the coalition are the same (homogeneous agents).

On the other hand, as established by Rotemberg (1994), in no case does Rational Altruism derive in degrees of effort consistent with the maximum efficiency solution levels. To obtain a Pareto efficient solution, all the Rational Altruism parameters should be equal to one ($\lambda_i = 1, \forall i$). However, the expression of the values of the $\lambda_i$ parameters [15] shows that at least one $\lambda_i$ will be strictly less than one.

Moreover, in the circumstances in which an agent has incentives to present a Rational Altruism parameter of over one, this is an adverse value effect. This can be shown simply by transforming expression [10] and expressing it in an equivalent manner:

\[
U_i^{RA} = (1 - \lambda_i) \times \left[ \frac{1}{N} \times \prod_{i=1}^{N} a_i^{\alpha_i} - [\sigma_i \times a_i] \right] + \lambda_i \times \left[ \prod_{i=1}^{N} a_i^{\alpha_i} - \sum_{j=1}^{N} [\sigma_j \times a_i] \right]
\]

(17)

The above expression shows that the greater the increments of $\lambda_i$ over one, the greater the divergence between the team member’s interest and the collective interest. Therefore, Rational Altruism parameter values of over one imply inefficiencies in collective terms. The analysed expression also shows that all values of the altruism
parameter within the range initially established by Rotemberg increase efficiency, compared to the alternative value of zero.

III. Second best solution for heterogeneous workers in the presence of Rational Altruism: an equity-based pay system.

The above section showed that Rational Altruism does not have an exclusively positive effect on business efficiency (in collective terms). This suggests the existence of an additional component to that established by Rotemberg, with a negative impact on business efficiency.

Therefore, in order to make full use of the value creation potential of Rational Altruism, we proceed to design an incentive system which favours the appearance of strategic altruistic behaviour in the sense proposed by Rotemberg, mitigating the negative effects detected in this paper (when $\lambda > 1$).

We seek a remuneration system ($R_i^\prime$) which provides incentives for agents to make a greater effort than they would be willing to make if the organisation’s output was equally distributed. The rest of the model’s hypotheses remain constant.

Therefore, the objective is to maximise the creation of wealth by the team, where the decision variable is the distribution of the total output among the members participating in its production. $R_i^\prime$ is the remuneration of each agent and it is defined as:

$$R_i^\prime = S_i \times F(a_1, \ldots, a_N);$$

where $S_i$ is the percentage of the total output received by agent $i$ as compensation for his contribution to the collective action. Moreover, $\sum_{i=1}^{N} S_i = 1$ must be true in order to maintain the constraint of no external subsidies. Finally, all agents must have incentives to participate ($S_i > 0, \quad \forall \ i$; and $R^\prime(Y) \geq C_i(a_i), \quad \forall \ i$).

The objective is to find the $S_i$ which maximises the wealth created by the collective action.

$$\max_{S_i} \left[ F(a_i(S_i, S_{-i}), a_{-i}(S_{-i}, S_i)) - \sum_{i=1}^{N} (\sigma_i \times a_i(S_i, S_{-i})) \right] \quad (18)$$
constrained to \( R'_i(Y) \geq C_i(a_i), \ \forall \ i \)

with \( \sum_{i=1}^{N} S_i = 1 \)

Therefore, each agent maximises his individual utility (\( U'_i \)):

\[
U'_i = S_i \times \left( \prod_{i=1}^{N} a_i \right) - \omega_i \times a_i \tag{19}
\]

and we obtain the Nash equilibrium degrees of effort pertaining to each individual (\( a_i^* \)):

\[
a_i^* = \left( \frac{S_i \times \alpha_i}{\omega_i} \right)^{\frac{1}{\sum \alpha_i}} \left( \prod_{j=1}^{1} \left( \frac{S_j \times \alpha_j}{\omega_j} \right)^{\frac{a_j}{\sum \alpha_j}} \right)^{\frac{a_i}{\sum \alpha_i}} \tag{20}
\]

Replacing \( a_i^* \) in the objective function (18), we obtain that the distribution rule \( (S_i^*) \) is:

\[
S_i^* = \frac{\alpha_i}{\sum_{i=1}^{N} \alpha_i} \tag{21}
\]

and it only depends on the relative elasticities of the degrees of effort. In order to differentiate it to the "equality based pay system" (\( R_i = Y/N \)) we denote this new rule \( (R'_i = S_i \times Y) \) as "equity based pay system", following Deckop’s (1995) nomenclature.

The role played by Rational Altruism is considered ex-post. Therefore, we then start with the maximisation of each agent’s individual utility, considering the presence of strategic altruism \( U^{'RA}_i \):

\[
Max U^{'RA}_i = \alpha_i \times \sum_{j \neq i} \left[ U'_j(a_j, a_{-j}) \right] + \lambda_i \times \sum_{j \neq i} \left[ U'_j(a_j, a_{-j}) \right] \tag{22}
\]

The solution to this problem shows that the degree of effort of each agent (\( a_i^{'RA*} \)) for given values of (\( \lambda_i \)) is:
Subsequently, each agent calculates the Rational Altruism value which maximises his individual utility, given effort $a_j^{\text{R.A}}$:

$$
Max_U^i = S_i^* \times \left( \prod_{i=1}^{N} (a_j^{\text{R.A}})^{\alpha_j} \right) - \sigma_i \times a_i^{\text{R.A}}
$$

(24)

The degree of Rational Altruism shown by each agent ($\lambda^*_i$) is thus:

$$
\lambda^*_i = \frac{\sum_{j=1}^{N} \alpha_j}{1 - \sum_{j=1}^{N} \alpha_j} \times \frac{S_i^*}{\sum_{j=1}^{N} S_j^*}
$$

(25)

Another alternative in the modelling process is to consider the role played by Rational Altruism ex-ante. The objective is to verify whether the solution is recursive or not and whether the suboptimal distribution rule is consistent with that obtained in the previous analysis.

We therefore calculate the equilibrium degree of effort for each agent in the presence of Rational Altruism ($a_i^{\text{R.A}}$):
Subsequently, from the previous expression \( a_{iR^A}^{*} \), we derive the problem of finding the rule for the distribution of the total output for each agent \( S_i^{RA} \), maximising the wealth generated by the organisation:

\[
\text{Max}_{S_i^{RA}} \left[ F(a_{iR^A}^{*} (S_i^{RA}, S_{-i}^{RA}), a_{-iR^A}^{*} (S_{-i}^{RA}, S_i^{RA})) - \sum_{i=1}^{N} (\sigma_i \times a_{iR^A}^{*} (S_i^{RA}, S_{-i}^{RA})) \right] 
\]

constrained to \( S_i^{RA} \times F(a_1, \ldots, a_n) \geq C_i(a_i) \)

with \( \sum_{i=1}^{N} S_i^{RA} = 1 \)

The sub-optimal distribution rule \( S_i^{R^A*} \) obtained is:

\[
S_i^{R^A*} = \frac{\alpha_i}{\sum_{j=1}^{N} \alpha_i} 
\]

As we can see, the expressions of \( S_i^{*} \) (21) and \( S_i^{R^A*} \) (28) have the same values. It is therefore confirmed that the solution is recursive and that the sub-optimal distribution rule obtained is independent of the presence of Rational Altruism or not.
The Rational Altruism parameter will adopt values from zero to one, regardless of whether or not there is symmetry between the elasticities of the agents’ productive factors. But, in the presence of asymmetries, the application of the new total output distribution rule increases the organisation’s efficiency measured in collective terms, both in relation to Rotemberg’s proposal (1994) and to the initial solution to the classic coalition problem.

Therefore, with $WCT (a_i, a_{-i})$ denoting the wealth generated for given degrees of effort and with $WCT^{**}$ to the optimal solution in terms of efficiency, it will always (with zero or negligible management costs) be true that:

$$WCT^{**} = WCT (a_i^{**}, a_{-i}^{**}) > WCT (a_i^{R+}, a_{-i}^{R+}) \geq WCT (a_i^{R_d}, a_{-i}^{R_d}) > WCT (a_i^*, a_{-i}^*)$$  \hspace{1cm} (29)

IV. Identification and assessment of Wicked Masked Altruism.

This section focuses on the identification and quantification of an additional component of Rational Altruism, other than that proposed by Rotemberg (1994). This additional component is manifest in agents who, with an equality-based distribution of production, present a Rational Altruism parameter of more than one. These agents are specifically those who present an elasticity of effort lower than the arithmetic mean of the elasticities of the other agents. This component, moreover, disappears when the distribution rule is based on a remuneration system depending on the relative elasticities of each of the agents.

To identify the nature of this additional component of Rational Altruism, we analyse the less productive agents. They have incentives to show self-serving solidarity in order to increase the effort of the most productive agents and, therefore, total output. However, the part of the increased output attributable to the marginal increase in effort of the less productive agents is smaller than the part attributable to the marginal increase in effort by the more productive agents. The less productive agents thus benefit from the productivity of their colleagues, as all the agents receive the same remuneration even though they do not equally participate in the process. Part of the self-serving manifestations of the less productive agents therefore mask conduct which attempts to benefit from the income of others.

The more productive agents, however, respond by reducing their efforts to defend
themselves from the movements of some of their colleagues. This defence movement means that their degree of Rational Altruism is less than would be expected in Rotemberg’s proposal (1994). This conduct leads to a less efficient final equilibrium solution, in collective terms. The loss of efficiency is due both to the decrease in the Rational Altruism of the more productive agents and the increase in that of the less productive agents (specifically, those with levels of over one).

To distinguish the motivation for Rational Altruism proposed by Rotemberg from the Rational Altruism derived from defensive/expropriation conduct, we refer to the latter as Wicked Masked Altruism.

An agent’s Wicked Masked Altruism is calculated through the difference between his degree of Rational Altruism when the total output is equally distributed and his degree of Rational Altruism when the total output is distributed according to the elasticities of effort. From the expression of both degrees of altruism, we obtain that Wicked Masked Altruism, \( \lambda_i^{WMA} \) adopts the following expression:

\[
\lambda_i^{WMA} = \frac{\sum_{j=1}^{N-1} \alpha_j}{N-1} - \frac{\alpha_i}{1 - \sum_{j=1}^{N-1} \alpha_j} = \frac{1}{N-1} \sum_{j=1}^{N-1} \alpha_j - \left( (N - 1) \times \alpha_i \right) \times \frac{1}{1 - \sum_{j=1}^{N-1} \alpha_j}
\]  

(30)

Wicked Masked Altruism adopts zero values in the case of equality of the elasticities of effort of all the agents; it has positive values (expropriation conduct) when the agents have elasticities of effort lower than the arithmetic mean of the elasticities of effort of their colleagues; and it is negative (defensive conduct) when the elasticities are higher than the arithmetic mean of the elasticities of their colleagues.

On the other hand, we see that as the number of agents participating in the collective action increases, there are less incentives for Wicked Masked Altruism to appear. This is because, when the output is distributed among more agents, the income to be obtained by each less productive agent is diluted.
V. Conclusions and business implications

This paper has focused on the role played by altruism in the efficiency of team production. Both the presence of an exogenous component in the utility function of agents (sociability) and the self-serving manifestations of altruism established by Rotemberg (Rational Altruism) could foster the appearance of cooperative behaviour among team members, obtaining more efficient equilibrium solutions in collective terms.

The Rational Altruism contemplated by Rotemberg is of a purely strategic nature. In this case, agents have incentives to show self-serving solidarity, even when they are selfish, as they thus increase their own utility. The final equilibrium situation leads to an increase in collective efficiency compared with a context in which the possibility of such self-serving conduct is not considered. The appearance of cooperative behaviour among the agents in a coalition was also detected by John Nash in his recent studies of the “Prisoner’s Dilemma” (Nash, 2005). In both cases, the increase in collective efficiency is based on the fact that agents respond to a repeated non-zero sum game. Nevertheless, manifestations of Rational Altruism do not lead to a Pareto efficient solution.

This paper, however, has shown that, in some circumstances, Rational Altruism has negative effects on business efficiency. This is the result of an additional component other than that postulated by Rotemberg. In this case, the conduct of the agents responds to a repeated zero-sum game of income expropriation (and defence). This additional component has been called Wicked Masked Altruism. The end result of its presence is a Nash equilibrium solution with loss of efficiency in collective terms, instead of what would be expected from Rotemberg’s proposal.

This paper also provides the design of a sub-optimal distribution rule which corrects the possible inefficiencies derived from Wicked Masked Altruism. The results show how the application of a payment system based on the relative elasticity of effort of each agent (equity-based pay system) does not provide incentives for the appearance of behaviour attempting to benefit from the income of others, and therefore for the appearance of defensive conduct. This payment system is independent of the consideration or not of the existence of Rational Altruism, as it is effective regardless.

The results obtained are applicable to all organisations applying organisation designs which foster team work and participative management systems. These systems
delegate decision-making power in people who have relevant information as a mechanism for fostering efficiency (Wruck and Jensen, 1994).

This type of organisation has two contractual alternatives when establishing efficiency-oriented design rules; on the one hand, the inclusion of a supervisor or administrator in the team (Alchian and Demsetz, 1972; Holmström, 1982), responsible for assessing performance (effort or total output) and paying the agents if they have not deviated from their efficient levels of effort. This hierarchy-based solution, which has zero residual losses, is associated with positive management costs in the form of the supervisor or administrator’s salary. Another option consists of establishing a pay system with zero management costs and increasing the efficiency of collective action by the agents’ strategic movements. It is specifically in the application of this non-hierarchy based (self-managed) contractual alternative where the results of this paper can be applied.

Lazear (1989) refers to a model concluding that the use of equality-based pay systems favours harmony between employees on the same hierarchical level. Otherwise, if each employee’s remuneration depends on relative comparisons with his colleagues (e.g. equity-based pay systems), there are incentives for non-cooperative behaviour which goes against the collective (firm’s) interests. Therefore, when designing a remuneration system to compensate the opportunity cost of the resources provided to the firm by each individual, the option of equally distributing the total output among the employees guarantees that they have no incentives to boycott their colleagues in order to improve their relative position and thus obtain greater remuneration.

An equality-based pay system, however, has negative effects, manifest through Wicked Masked Altruism, when employees are not equally productive. In this case, the equity-based pay system makes the most of the potential ability of Rational Altruism to reduce the residual losses generated in the coalition model. Moreover, if the final residual loss is less than the salary of the team supervisor or administrator, the self-managed option would be better, in terms of efficiency, than the hierarchy-based option proposed by Alchian and Demsetz (1972) or Holmström (1982).
VI. Bibliographical references


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